Exercise 16

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_0^{x^4} \cos^2\theta \, d\theta$$

Solution

According to part 1 of the fundamental theorem of calculus,

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

In order to make the upper limit a single variable, let $u = x^4$.

$$y = \int_0^u \cos^2\theta \, d\theta$$

As a result, using the chain rule,

$$\begin{aligned} y' &= \frac{d}{dx}\int_0^u \cos^2\theta \, d\theta \\ &= \frac{du}{dx}\frac{d}{du}\int_0^u \cos^2\theta \, d\theta \\ &= \frac{du}{dx}\cos^2 u \\ &= 4x^3\cos^2 x^4. \end{aligned}$$